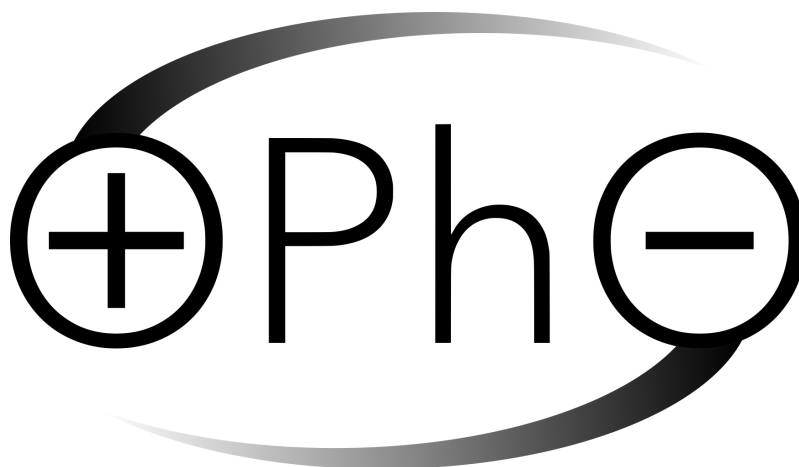


# 2022 Online Physics Olympiad: Open Contest



## Sponsors

This competition could not be possible without the help of our sponsors, who are all doing great things in physics, math, and education.



Jane Street



Wolfram  
Language™



AwesomeMath  
*making x,y,z as easy as a, b, c*



## Instructions

If you wish to request a clarification, please use [this form](#). To see all clarifications, see [this document](#).

- Use  $g = 9.81 \text{ m/s}^2$  in this contest, **unless otherwise specified**. See the constants sheet on the following page for other constants.
- This test contains 35 short answer questions. Each problem will have three possible attempts.
- The weight of each question depends on our scoring system found [here](#). Put simply, the later questions are worth more, and the overall amount of points from a certain question decreases with the number of attempts that you take to solve a problem as well as the number of teams who solve it.
- Any team member is able to submit an attempt. Choosing to split up the work or doing each problem together is up to you. Note that after you have submitted an attempt, your teammates must refresh their page before they are able to see it.
- Answers should contain **three** significant figures, unless otherwise specified. All answers within the 1% range will be accepted.
- When submitting a response using scientific notation, please use exponential form. In other words, if your answer to a problem is  $A \times 10^B$ , please type  $AeB$  into the submission portal.
- A standard scientific or graphing handheld calculator *may* be used. Technology and computer algebra systems like Wolfram Alpha or the one in the TI nSpire will not be needed or allowed. Attempts to use these tools will be classified as cheating.
- You are *allowed* to use Wikipedia or books in this exam. Asking for help on online forums or your teachers will be considered cheating and may result in a possible ban from future competitions.
- Top scorers from this contest will qualify to compete in the Online Physics Olympiad *Invitational Contest*, which is an olympiad-style exam. More information will be provided to invitational qualifiers after the end of the *Open Contest*.
- In general, answer in SI units (meter, second, kilogram, watt, etc.) unless otherwise specified. Please input all angles in degrees unless otherwise specified.
- If the question asks to give your answer as a percent and your answer comes out to be “ $x\%$ ”, please input the value  $x$  into the submission form.
- Do not put units in your answer on the submission portal! If your answer is “ $x$  meters”, input only the value  $x$  into the submission portal.
- **Do not communicate information to anyone else apart from your team-members before June 13, 2022.**

## List of Constants

- Proton mass,  $m_p = 1.67 \cdot 10^{-27}$  kg
- Neutron mass,  $m_n = 1.67 \cdot 10^{-27}$  kg
- Electron mass,  $m_e = 9.11 \cdot 10^{-31}$  kg
- Avogadro's constant,  $N_0 = 6.02 \cdot 10^{23}$  mol<sup>-1</sup>
- Universal gas constant,  $R = 8.31$  J/(mol · K)
- Boltzmann's constant,  $k_B = 1.38 \cdot 10^{-23}$  J/K
- Electron charge magnitude,  $e = 1.60 \cdot 10^{-19}$
- 1 electron volt,  $1 \text{ eV} = 1.60 \cdot 10^{-19}$  J
- Speed of light,  $c = 3.00 \cdot 10^8$  m/s
- Universal Gravitational constant,

$$G = 6.67 \cdot 10^{-11} \text{ (N} \cdot \text{m}^2\text{)/kg}^2$$

- Solar Mass

$$M_{\odot} = 1.988 \cdot 10^{30} \text{ kg}$$

- Acceleration due to gravity,  $g = 9.81$  m/s<sup>2</sup>
- 1 unified atomic mass unit,

$$1 \text{ u} = 1.66 \cdot 10^{-27} \text{ kg} = 931 \text{ MeV}/c^2$$

- Planck's constant,

$$h = 6.63 \cdot 10^{-34} \text{ J} \cdot \text{s} = 4.41 \cdot 10^{-15} \text{ eV} \cdot \text{s}$$

- Permittivity of free space,

$$\epsilon_0 = 8.85 \cdot 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2)$$

- Coulomb's law constant,

$$k = \frac{1}{4\pi\epsilon_0} = 8.99 \cdot 10^9 \text{ (N} \cdot \text{m}^2\text{)/C}^2$$

- Permeability of free space,

$$\mu_0 = 4\pi \cdot 10^{-7} \text{ T} \cdot \text{m/A}$$

- Magnetic constant,

$$\frac{\mu_0}{4\pi} = 1 \cdot 10^{-7} \text{ (T} \cdot \text{m)/A}$$

- 1 atmospheric pressure,

$$1 \text{ atm} = 1.01 \cdot 10^5 \text{ N/m}^2 = 1.01 \cdot 10^5 \text{ Pa}$$

- Wien's displacement constant,  $b = 2.9 \cdot 10^{-3} \text{ m} \cdot \text{K}$

- Stefan-Boltzmann constant,

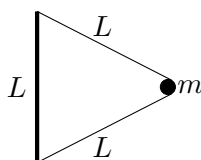
$$\sigma = 5.67 \cdot 10^{-8} \text{ W/m}^2/\text{K}^4$$

## Problems

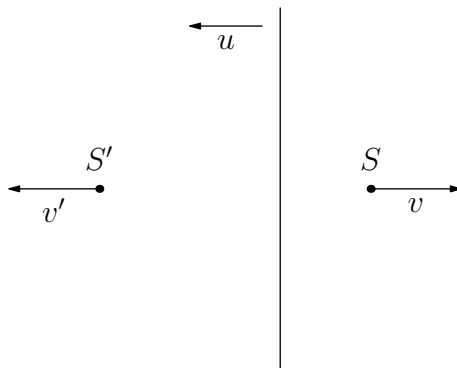
**1. TWO PROJECTILES** A player throws two tennis balls on a level ground at  $v = 20$  m/s in the same direction, once at an angle of  $\alpha = 35^\circ$  and once at an angle  $\beta = 55^\circ$  to the horizontal. The distance between the landing spots of the two balls is  $d$ . Find  $d$  in meters.

Assume the height of the player is negligible and ignore air resistance.

**2. BOW AND ARROW** Consider the following simple model of a bow and arrow. An ideal elastic string has a spring constant  $k = 10$  N/m and relaxed length  $L = 1$  m which is attached to the ends of an inflexible fixed steel rod of the same length  $L$  as shown below. A small ball of mass  $m = 2$  kg and the thread are pulled by its midpoint away from the rod until each individual part of the thread have the same length of the rod, as shown below. What is the speed of the ball in meters per seconds right after it stops accelerating? Assume the whole setup is carried out in zero gravity.



**3. CITY LIGHTS** A truck (denoted by  $S$ ) is driving at a speed  $v = 2$  m/s in the opposite direction of a car driving at a speed  $u = 3$  m/s, which is equipped with a rear-view mirror. Both  $v$  and  $u$  are measured from an observer on the ground. Relative to this observer, what is the speed (in m/s) of the truck's image  $S'$  through the car's mirror? Car's mirror is a plane mirror.



**4. SPRINGING EARTH** For this problem, assume the Earth moves in a perfect circle around the sun in the  $xy$  plane, with a radius of  $r = 1.496 \times 10^{11}$  m, and the Earth has a mass  $m = 5.972 \times 10^{24}$  kg. An alien stands far away from our solar system on the  $x$  axis such that it appears the Earth is moving along a one dimensional line, as if there was a zero-length spring connecting the Earth and the Sun.

For the alien at this location, it is impossible to tell just from the motion if it's 2D motion via gravity or 1D motion via a spring. Let  $U_g$  be the gravitational potential energy ignoring its self energy if Earth moves via gravity, taking potential energy at infinity to be 0 and  $U_s$  be the maximum spring potential energy if Earth moves in 1D via a spring. Compute  $U_g/U_s$ .

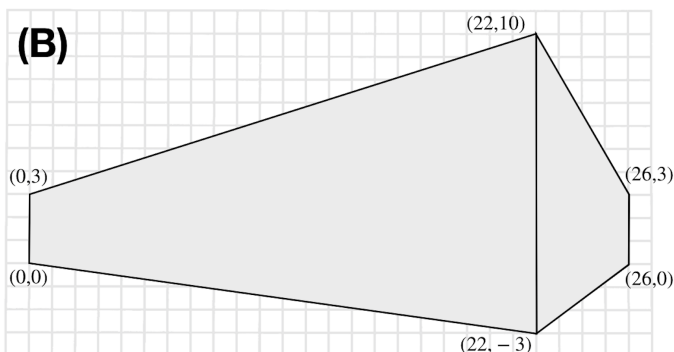
**5. BATTLE ROPES** Battle ropes can be used as a full body workout (see photo). It consists of a long piece of thick rope (ranging from 35 mm to 50 mm in diameter), wrapped around a stationary pole. The athlete grabs on to both ends, leans back, and moves their arms up and down in order to create waves, as shown in the photo.



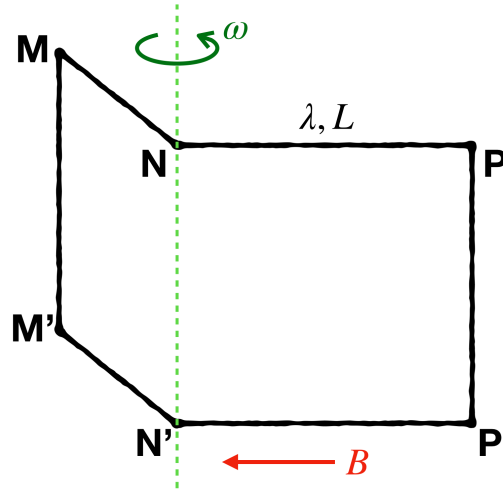
The athlete wishes to upgrade from using a 35 mm diameter rope to a 50 mm diameter rope, while keeping everything else the same (rope material, rope tension, amplitude, and speed at which her arms move back and forth). By doing so, the power she needs to exert changes from  $P_0$  to  $P_1$ . Compute  $P_1/P_0$ .

**6. POLARIZERS** Given vertically polarized light, you're given the task of changing it to horizontally polarized light by passing it through a series of  $N = 5$  linear polarizers. What is the maximum possible efficiency of this process? (Here, efficiency is defined as the ratio between output light intensity and input light intensity.)

**7. FATAL FRAME** These days, there are so many stylish rectangular home-designs (see figure A). It is possible from the outline of those houses in their picture to estimate with good precision where the camera was. Consider an outline in one photograph of a rectangular house which has height  $H = 3$  meters (see figure B for square-grid coordinates). Assume that the camera size is negligible, how high above the ground (in meters) was the camera at the moment this picture was taken?



**8. THE WIRE** Consider a thin rigid wire-frame  $MNPP'N'M'$  in which  $MNN'M'$  and  $NPP'N'$  are two squares of side  $L$  with resistance per unit-length  $\lambda$  and their planes are perpendicular. The frame is rotated with a constant angular velocity  $\omega$  around an axis passing through  $NN'$  and put in a region with constant magnetic field  $B$  pointing perpendicular to  $NN'$ . What is the total heat released on the frame per revolution (in Joules)? Use  $L = 1\text{ m}$ ,  $\lambda = 1\Omega/\text{m}$ ,  $\omega = 2\pi\text{ rad/s}$  and  $B = 1\text{ T}$ .



**9. MELTING ICEBERG** In this problem, we explore how fast an iceberg can melt, through the dominant mode of forced convection. For simplicity, consider a very thin iceberg in the form of a square with side lengths  $L = 100\text{ m}$  and a height of  $1\text{ m}$ , moving in the arctic ocean at a speed of  $0.2\text{ m/s}$  with one pair of edges parallel to the direction of motion (Other than the height, these numbers are typical of an average iceberg). The temperature of the surrounding water and air is  $2^\circ\text{C}$ , and the temperature of the iceberg is  $0^\circ\text{C}$ . The density of ice is  $917\text{ kg/m}^3$  and the latent heat of melting is  $L_w = 334 \times 10^3\text{ J/kg}$ .

The heat transfer rate  $\dot{Q}$  between a surface and the surrounding fluid is dependent on the heat transfer coefficient  $h$ , the surface area in contact with the fluid  $A$ , and the temperature difference between the surface and the fluid  $\Delta T$ , via  $\dot{Q} = hA\Delta T$ .

In heat transfer, three useful quantities are the Reynold's number, the Nusselt number, and the Prandtl number. Assume they are constant through and given by (assuming laminar flow):

$$\text{Re} = \frac{\rho v_\infty L}{\mu}, \quad \text{Nu} = \frac{hL}{k}, \quad \text{Pr} = \frac{c_p \mu}{k}$$

where:

- $\rho$ : density of the fluid
- $v_\infty$ : speed of the fluid with respect to the object (at a very far distance)
- $L$ : length of the object in the direction of motion
- $\mu$ : dynamic viscosity of the fluid
- $k$ : thermal conductivity of the fluid
- $c_p$ : the specific heat capacity of the fluid

Through experiments, the relationship between the three dimensionless numbers is, for a flat plate:

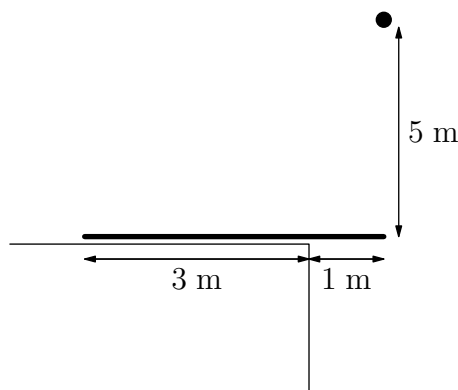
$$\text{Nu} = 0.664\text{Re}^{1/2}\text{Pr}^{1/3}.$$

Use the following values for calculations:

	Air	Water
$\rho$ (kg/m <sup>3</sup> )	1.29	1000
$\mu$ (kg/(m · s))	$1.729 \times 10^{-5}$	$1.792 \times 10^{-3}$
$c_p$ (J/(kg · K))	1004	4220
$k$ (W/(m · K))	0.025	0.556

The initial rate of heat transfer is  $\dot{Q}$ . Assuming this rate is constant (this is not true, but will allow us to obtain an estimate), how long (in days) would it take for the ice to melt completely? Assume convection is only happening on the top and bottom faces. Round to the nearest day.

**10. SCALE** A scale of uniform mass  $M = 3$  kg of length  $L = 4$  m is kept on a rough table (infinite friction) with  $l = 1$  m hanging out of the table as shown in the figure below. A small ball of mass  $m = 1$  kg is released from rest from a height of  $h = 5$  m above the end of the scale. Find the maximum angle (in degrees) that the scale rotates by in the subsequent motion if ball sticks to the scale after collision. Take gravity  $g = 10$  m/s<sup>2</sup>.



**11. LEVITATING** In a galaxy far, far away, there is a planet of mass  $M = 6 \cdot 10^{27}$  kg which is a sphere of radius  $R$  and charge  $Q = 10^3$  C uniformly distributed. Aliens on this planet have devised a device for transportation, which is an insulating rectangular plate with mass  $m = 1$  kg and charge  $q = 10^4$  C. This transportation device moves in a circular orbit at a distance  $r = 8 \cdot 10^6$  m from the center of the planet. The aliens have designated this precise elevation for the device, and do not want the device to deviate at all. In order to maintain its orbit, the device contains a relatively small energy supply. Find the power (in Watts) that the energy supply must release in order to sustain this orbit.

The velocity of the device can be assumed to be much smaller than the speed of light, so that relativistic effects can be ignored. The device can also be assumed to be small in comparison to the size of the planet.

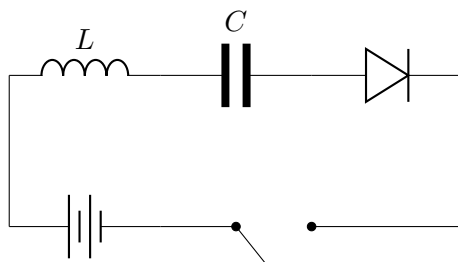
**12. SINGING IN THE RAIN** A raindrop of mass  $M = 0.035$  g is at height  $H = 2$  km above a large lake. The raindrop then falls down (without initial velocity), mixing and coming to equilibrium with the lake. Assume that the raindrop, lake, air, and surrounding environment are at the same temperature  $T = 300$  K. Determine the magnitude of entropy change associated with this process (in J/K).

**13. ROCKET LAUNCH** A rocket with mass of 563.17 (not including the mass of fuel) metric tons sits on the launchpad of the Kennedy Space Center (latitude  $28^{\circ}31'27''\text{N}$ , longitude  $80^{\circ}39'03''\text{W}$ ), pointing directly upwards. Two solid fuel boosters, each with a mass of 68415kg and providing 3421kN of thrust are pointed directly downwards.

The rocket also has a liquid fuel engine, that can be throttled to produce different amounts of thrust and gimbaled to point in various directions. What is the minimum amount of thrust, in kN, that this engine needs to provide for the rocket to lift vertically (to accelerate directly upwards) off the launchpad?

Assume  $G = 6.674 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{s}^2}$ , and that the Earth is a perfect sphere of radius 6370km and mass  $5.972 \times 10^{24}\text{kg}$  that completes one revolution every 86164s and that the rocket is negligibly small compared to the Earth. Ignore buoyancy forces.

**The following information applies for the next two problems.** A circuit has a power source of  $\mathcal{E} = 5.82\text{V}$  connected to three elements in series: an inductor with  $L = 12.5\text{mH}$ , a capacitor with  $C = 48.5\mu\text{F}$ , and a diode with threshold voltage  $V_0 = 0.65\text{V}$ . (Of course, the polarity of the diode is aligned with that of the power source.) You close the switch, and after some time, the voltage across the capacitor becomes constant. (*Note:* An ideal diode with threshold voltage  $V_0$  is one whose IV characteristic is given by  $I = 0$  for  $V < V_0$  and  $V = V_0$  for  $I > 0$ .)

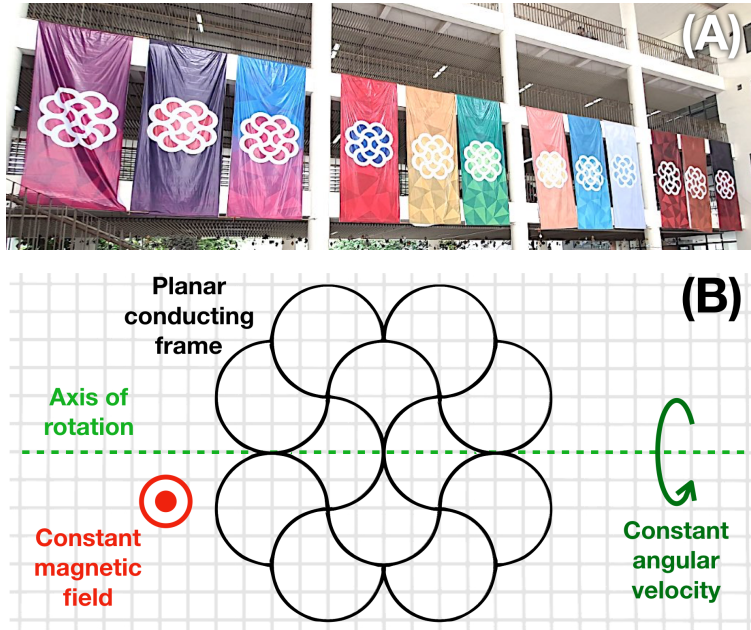


**14. LC-DIODE 1** How much time (in seconds) has elapsed before the voltage across the capacitor becomes constant?

**15. LC-DIODE 2** What is the magnitude of final voltage (in volts) across the capacitor?



**16. RAGING LOOP** At Hanoi-Amsterdam High School in Vietnam, every subject has its own flag (see Figure A, taken by Tung X. Tran). While the flags differ in color, they share the same central figure. Consider a planar conducting frame of that figure rotating at a constant angular velocity in a uniform magnetic field (see Figure B). The frame is made of thin rigid wires with uniform curvature and resistance per unit length. What fraction of the total heat released is released by the outermost wires?

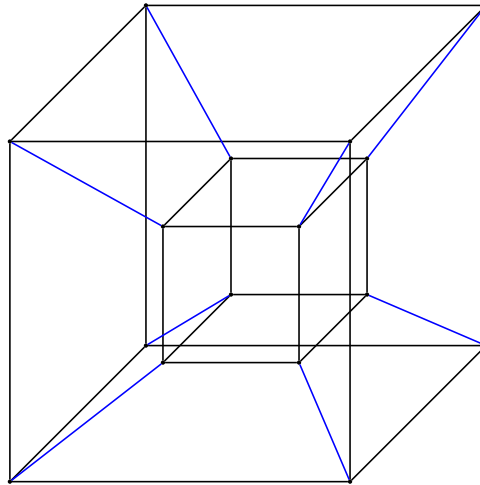


**17. MOON LANDING** A spacecraft is orbiting in a very low circular orbit at a velocity  $v_0$  over the equator of a perfectly spherical moon with uniform density. Relative to a stationary frame, the spacecraft completes a revolution of the moon every 90 minutes, while the moon revolves in the same direction once every 24 hours. The pilot of the spacecraft would like to land on the moon using the following process:

1. Start by firing the engine directly against the direction of motion.
2. Orient the engine over time such that the vertical velocity of the craft remains 0, while the horizontal speed continues to decrease.
3. Once the velocity of the craft relative to the ground is also 0, turn off the engine.

Assume that the engine of the craft can be oriented instantly in any direction, and the craft has a TWR (thrust-to-weight ratio, where weight refers to the weight at the moon's surface) of 2, which remains constant throughout the burn. If the craft starts at  $v_0 = 500$  m/s, compute the delta-v expended to land, minus the initial velocity, i.e.  $\Delta v - v_0$ .

**18. TESSERACT OSCILLATIONS** A tesseract is a 4 dimensional example of cube. It can be drawn in 3 dimensions by drawing two cubes and connecting their vertices together as shown in the picture below:



Now for the 3D equivalent. The lines connecting the vertices are replaced with ideal springs of constant  $k = 10 \text{ N/m}$  (in blue in the figure). Now, suppose the setup is placed in zero-gravity and the outer cube is fixed in place with a sidelength of  $b = 2 \text{ m}$ . The geometric center of the inner cube is placed in the geometric center of the outer cube, and the inner cube has a side-length  $a = 1 \text{ m}$  and mass  $m = 1.5 \text{ kg}$ . The inner cube is slightly displaced from equilibrium. Consider the period of oscillations

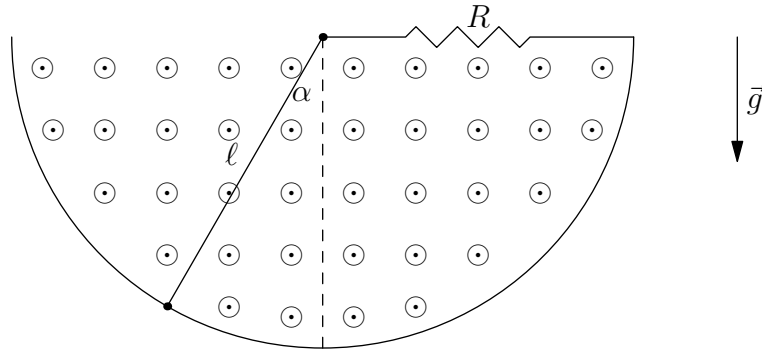
- $T_1$ : when the springs have a relaxed length of 0;
- $T_2$ : when the springs are initially relaxed before the inner cube is displaced.

What is  $T_1 + T_2$ ?

**19. THE ROOM** Consider two points  $S$  and  $S'$  randomly placed inside a  $D$ -dimensional [hyper-rectangular](#) room with walls that are perfect-reflecting  $(D - 1)$ -dimensional hyper-plane mirrors. How many different light-rays that start from  $S$ , reflect  $N$  times on one of the walls and  $N - 1$  times on each of the rest, then go to  $S'$ ? Use  $D = 7$  and  $N = 3$ .

**20. TWO RINGS** Two concentric isolated rings of radius  $a = 1 \text{ m}$  and  $b = 2 \text{ m}$  of mass  $m_a = 1 \text{ kg}$  and  $m_b = 2 \text{ kg}$  are kept in a gravity free region. A soap film of surface tension  $\sigma = 0.05 \text{ Nm}^{-1}$  with negligible mass is spread over the rings such that it occupies the region between the rings. The smaller ring is pulled slightly along the axis of the rings. Find the time period of small oscillation in seconds.

**21. PENDULUM CIRCUIT** An open electrical circuit contains a wire loop in the shape of a semi-circle, that contains a resistor of resistance  $R = 0.2\Omega$ . The circuit is completed by a conducting pendulum in the form of a uniform rod with length  $\ell = 0.1$  m and mass  $m = 0.05$  kg, has no resistance, and stays in contact with the other wires at all times. All electrical components are oriented in the  $yz$  plane, and gravity acts in the  $z$  direction. A constant magnetic field of strength  $B = 2$  T is applied in the  $+x$  direction.

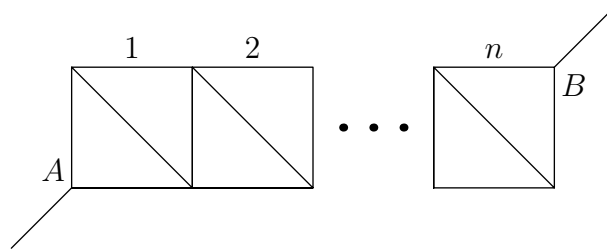


Ignoring self inductance and assuming that  $\alpha \ll 1$ , the general equation of motion is in the form of  $\theta(t) = A(t) \cos(\omega t + \varphi)$ , where  $A(t) \geq 0$ . Find  $\omega^2$ .

**22. BROKEN TABLE** A table of unknown material has a mass  $M = 100$  kg, width  $w = 4$  m, length  $\ell = 3$  m, and 4 legs of length  $L = 0.5$  m with a Young's modulus of  $Y = 1.02$  MPa at each of the corners. The cross-sectional area of a table leg is approximately  $A = 1$  cm<sup>2</sup>. The surface of the table has a coefficient of friction of  $\mu = 0.1$ . A point body with the same mass as the table is put at some position from the geometric center of the table. What is the minimum distance the body must be placed from the center such that it slips on the table surface immediately after? Report your answer in centimeters.

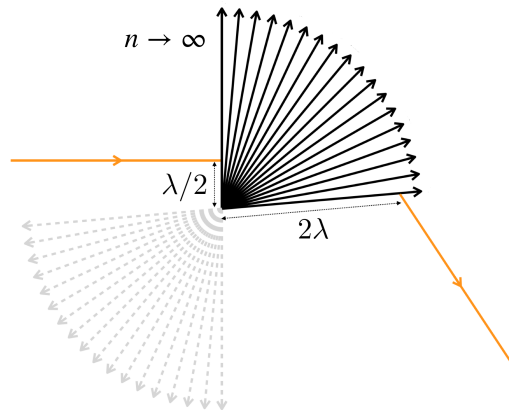
The table surface and floor are non-deformable.

**23. RESISTANCE BOX** In the figure below, the resistance of each wire (side and diagonal) is  $1\Omega$ . Find the value of  $p + q$  if  $\lim_{n \rightarrow \infty} \frac{R_{AB}}{n} = \frac{p}{q}$  where  $p$  and  $q$  are co-prime integers.



**24. DIPOLE CONDUCTOR** An (ideal) electric dipole of magnitude  $p = 1 \times 10^{-6}$  C·m is placed at a distance  $a = 0.05$  m away from the center of an uncharged, isolated spherical conductor of radius  $R = 0.02$  m. Suppose the angle formed by the dipole vector and the radial vector (the vector pointing from the sphere's center to the dipole's position) is  $\theta = 20^\circ$ . Find the (electrostatic) interaction energy between the dipole and the charge induced on the spherical conductor.

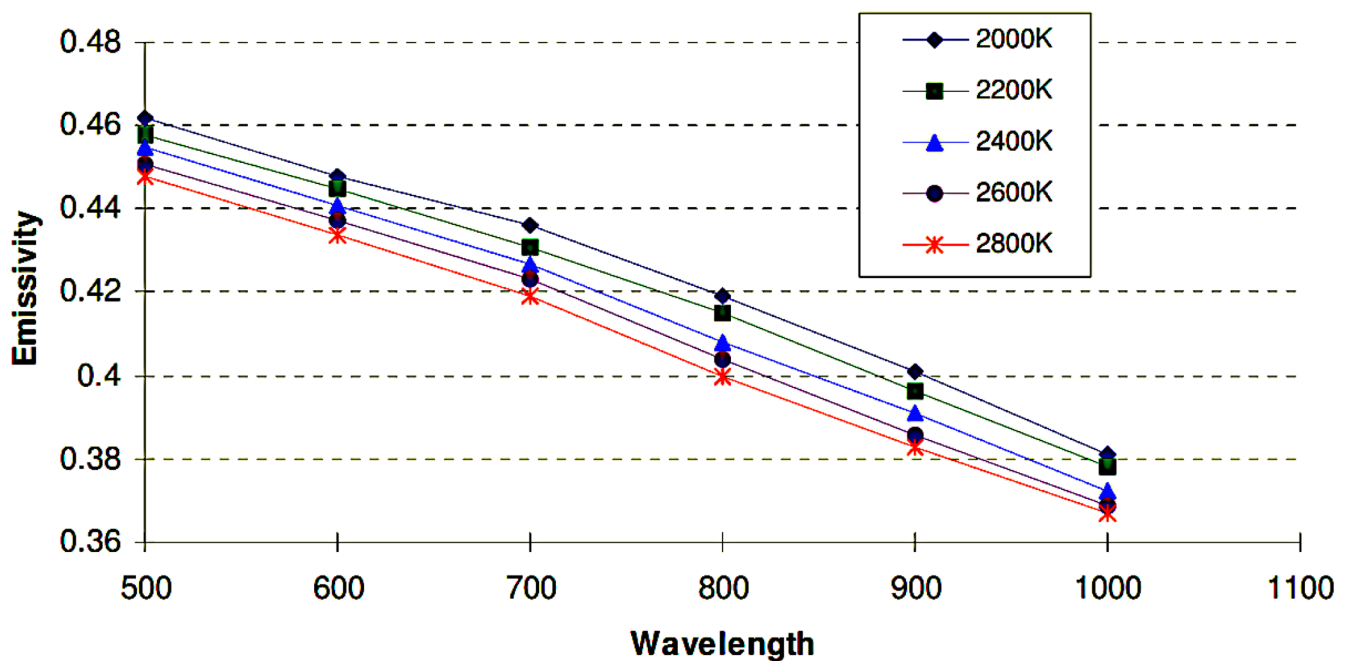
**25. DYING LIGHT** Consider an optical system made of many identical ideal (negligible-thickness) half-lenses with focal length  $f > 0$ , organized so that they share the same center and are angular-separated equally at density  $n$  (number of lenses per unit-radian). Define the length-scale  $\lambda = f/n$ . A light-ray arrives perpendicular to the first lens at distance  $\lambda/2$  away from the center, then leaves from the last lens at distance  $2\lambda$  away from the center. Estimate the total deflection angle (in rad) of the light-ray by this system in the limit  $n \rightarrow \infty$ .



**26. TUNGSTEN** For black body radiation, Wien's Displacement Law states that its spectral radiance will peak at

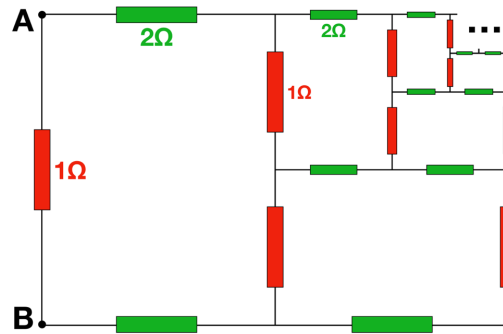
$$\lambda_{\text{peak}} = \frac{b}{T}.$$

where  $b = 2.89777 \times 10^{-3}$  mK, and  $T$  is the temperature of the object. When QiLin tried to reproduce this in a lab, by working with a tungsten-filament lightbulb at 2800 K, he computed a different value for  $b$  by measuring the peak wavelength using a spectrometer and multiplying it with the temperature. He hypothesizes that this discrepancy is because tungsten is not an ideal black body. The graph below, courtesy of the CRC Handbook of Chemistry and Physics, shows the emissivity of tungsten at various conditions (the units for wavelength is nm).



Assuming QiLin's hypothesis is correct, and assuming there were no other errors in the experiment, how off was his value for  $b$ ? Submit  $\frac{|b_{\text{theory}} - b_{\text{experiment}}|}{b_{\text{theory}}}$  as a decimal number, to *one* significant digit (giving you room to estimate where the points are).

**27. BIOSHOCK INFINITE** The equivalent resistance (in  $\Omega$ ) between points A and B of the following infinite resistance network made of  $1\Omega$  and  $2\Omega$  resistors is  $0.abcdefgh...$  in decimal form. Enter  $efg$  into the answer box (It should be an integer in the range of 0-999).



**28. MAGNETIC BALL** A uniform spherical metallic ball of mass  $m$ , resistivity  $\rho$ , and radius  $R$  is kept on a smooth friction-less horizontal ground. A horizontal uniform, constant magnetic field  $B$  exists in the space parallel to the surface of ground. The ball was suddenly given an impulse perpendicular to magnetic field such that ball begin to move with velocity  $v$  without losing the contact with ground. Find the time in seconds required to reduce its velocity by half.

Numerical Quantities:  $m = 2$  kg,  $4\pi\epsilon_0 R^3 B^2 = 3$  kg,  $\rho = 10^9 \Omega\text{m}$ ,  $v = \pi$  m/s.

**For the following two problems, this information applies.** Assume  $g = 9.8\text{m/s}^2$ . On a balcony, a child holds a spherical balloon of radius 15 cm. Upon throwing it downwards with a velocity of 4.2 m/s, the balloon starts magically expanding, its radius increasing at a constant rate of 35 cm/s. Another child, standing on the ground, is holding a hula hoop, 4 m below the point where the center of the balloon was released.

**29. MAGICAL BALLOON 1** If the minimum radius of the hoop such that the balloon falls completely through the hula hoop without touching it is  $r$ , compute the difference between  $r$  and the largest multiple of 5cm less than or equal to  $r$ . Answer in centimeters; your answer should be in the range  $[0, 5)$ .

**30. MAGICAL BALLOON 2** Consider the horizontal plane passing through the center of the balloon at the start. If the total volume above this plane that the balloon falls through after it is thrown downwards is  $V$ , compute the difference between  $V$  and half the original volume of the balloon. Answer in milliliters; your answer should be nonnegative.

Note that when refer to the “volume an object falls through”, it refers to the volume of the union of all points in space which the object occupies as it falls.

**31. HYDROGEN MAGNETISM** In quantum mechanics, when calculating the interaction between the electron with the proton in a hydrogen atom, it is necessary to compute the following volume integral (over all space):

$$\mathbf{I} = \int \mathbf{B}(\mathbf{r}) |\Psi(\mathbf{r})|^2 dV$$

where  $\Psi(\mathbf{r})$  is the spatial wavefunction of the electron as a function of position  $\mathbf{r}$  and  $\mathbf{B}(\mathbf{r})$  is the (boldface denotes vector) magnetic field produced by the proton at position  $\mathbf{r}$ . Suppose the proton is located at the origin and it acts like a finite-sized magnetic dipole (but much smaller than  $a_0$ ) with dipole moment  $\mu_p = 1.41 \times 10^{-26}$  J/T. Let the hydrogen atom be in the ground state, meaning  $\Psi(\mathbf{r}) = \frac{e^{-r/a_0}}{\sqrt{\pi a_0^3}}$ , where  $a_0 = 5.29 \times 10^{-11}$  m is the Bohr radius. Evaluate the magnitude of the integral  $|\mathbf{I}|$  (in SI units).

**32. RELATIVISTIC COLLISION** Zed is trying to model the repulsive interaction between 2 objects,  $A$  and  $B$  (with masses  $m_A$  and  $m_B$ , respectively), in a relativistic setting. He knows that in relativity, forces cannot act at a distance, so he models the repulsive force with a small particle of mass  $m$  that bounces elastically between  $A$  and  $B$ . Throughout this problem, assume everything moves on the  $x$ -axis. Suppose that initially,  $A$  and  $B$  have positions and velocities  $x_A, v_A$  and  $x_B, v_B$ , respectively, where  $x_A < x_B$  and  $v_A > v_B$ . The particle has an initial (relativistic) speed  $v$ .

For simplicity, assume that the system has no total momentum. You may also assume that  $v_A, v_B \ll v$ , and that  $p_m \ll p_A, p_B$ , where  $p_m, p_A, p_B$  are the momenta of the particle,  $A$ , and  $B$ , respectively. Do NOT assume  $v \ll c$ , where  $c$  is the speed of light.

Find the position (in m) of  $A$  when its velocity is 0, given that  $m_A = 1$  kg,  $m_B = 2$  kg,  $v_A = 0.001c$ ,  $m = 1 \times 10^{-6}$  kg,  $v = 0.6c$ ,  $x_A = 0$  m,  $x_B = 1000$  m.

*Note:* Answers will be tolerated within 0.5%, unlike other problems.

**33. MICROSCOPE** Consider an optical system consisting of two thin lenses sharing the same optical axis. When a cuboid with a side parallel to the optical axis is placed to the left of the left lens, its final image formed by the optical system is also a cuboid but with 500 times the original volume. Assume the two lenses are 10 cm apart and such a cuboid of volume  $1 \text{ cm}^3$  is placed such that its right face is 2 cm to the left of the left lens. What's the maximum possible volume of the intermediate image (i.e., image formed by just the left lens) of the cuboid? Answer in  $\text{cm}^3$ .

**34. RESISTOR GRID** Consider an infinite square grid of equal resistors where the nodes are exactly the lattice points in the 2D Cartesian plane. A current  $I = 2.7$  A enters the grid at the origin  $(0,0)$ . Find the current in Amps through the resistor connecting the nodes  $(N,0)$  and  $(N,1)$ , where  $N = 38$  can be assumed to be much larger than 1.

**35. STRANGE GAS** Suppose we have a non-ideal gas, and in a certain volume range and temperature range, it is found to satisfy the state relation

$$p = AV^\alpha T^\beta$$

where  $A$  is a constant,  $\alpha = -\frac{4}{5}$  and  $\beta = \frac{3}{2}$ , and the other variables have their usual meanings. Throughout the problem, we will assume to be always in that volume and temperature range.

Assume that  $\gamma = \frac{C_p}{C_v}$  is found to be constant for this gas ( $\gamma$  is independent of the state of the gas), where  $C_p$  and  $C_v$  are the heat capacities at constant pressure and volume, respectively. What is the minimum possible value for  $\gamma$ ?