# 2025 Online Physics Olympiad: Invitational Contest

# Part I

## Sponsors

This competition could not be possible without the help of our sponsors, who are all doing great things in physics, math, and education.

## Instructions

Part 1 of the Invitational Round consists of three theoretical free response questions over 2 full days from August 23, 12:00 AM UTC to August 24, 11:59 PM UTC. Part 2 consists of a fourth theoretical question and one experimental problem, to be completed during the second day, from August 24, 12:00 AM UTC to August 24, 11:59 PM UTC.

- **The team leader should submit their final solution document in this google form. We don't anticipate a tie, but in the rare circumstance that there is one, the time you submit will be used to break it.** You are allowed to submit up 10 MB of data for each problem solution. It is recommended that participants write their solutions in LaTeX. However, handwritten solutions (or a combination of both) are accepted too. If participants have more than one photo of a handwritten solution (`jpg, png`, etc), it is required to organize them in the correct order in a `pdf` before submitting. If you wish a premade LaTeX template, we have made one for you here.

- If you wish to request a clarification, please use this form. To see all clarifications, view this document.

- Since each question is a long answer response, participants will be judged on the quality of their work. To receive full points, participants need to show their work, including deriving equations. As a general rule of thumb, any common equations (such as the ones in the IPhO formula sheet) can be cited without proof.

- Remember to state any approximations made and which system of equations were solved after every step. Explicitly showing every step of algebra is not necessary. Participants may leave all final answers in symbolic form (in terms of variables) unless otherwise specified. Be sure to state all assumptions.

- Participants are required to keep all their rough working for both parts of this round and to submit it to this form by 30 minutes after the end of the competition, i.e. the deadline is August 25, 12:30 AM UTC. Note that we will not explicitly request working from any team: all teams are required to submit their rough working.

## List of Constants

- Proton mass, $m_p = 1.67 \cdot 10^{-27}$ kg
- Neutron mass, $m_n = 1.67 \cdot 10^{-27}$ kg
- Electron mass, $m_e = 9.11 \cdot 10^{-31}$ kg
- Avogadro's constant, $N_0 = 6.02 \cdot 10^{23}$ mol$^{-1}$
- Universal gas constant, $R = 8.31$ J/(mol $\cdot$ K)
- Boltzmann's constant, $k_B = 1.38 \cdot 10^{-23}$ J/K
- Electron charge magnitude, $e = 1.60 \cdot 10^{-19}$ C
- 1 electron volt, 1 eV $= 1.60 \cdot 10^{-19}$ J
- Speed of light, $c = 3.00 \cdot 10^8$ m/s
- Universal Gravitational constant,
$$G = 6.67 \cdot 10^{-11} \text{ (N} \cdot \text{m}^2)/\text{kg}^2$$
- Solar Mass
$$M_\odot = 1.988 \cdot 10^{30} \text{ kg}$$
- Acceleration due to gravity, $g = 9.8$ m/s$^2$
- 1 unified atomic mass unit,
$$1 \text{ u} = 1.66 \cdot 10^{-27} \text{ kg} = 931 \text{ MeV/c}^2$$
- Planck's constant,
$$h = 6.63 \cdot 10^{-34} \text{ J} \cdot \text{s} = 4.41 \cdot 10^{-15} \text{ eV} \cdot \text{s}$$
- Permittivity of free space,
$$\epsilon_0 = 8.85 \cdot 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2)$$
- Coulomb's law constant,
$$k = \frac{1}{4\pi\epsilon_0} = 8.99 \cdot 10^9 \text{ (N} \cdot \text{m}^2)/\text{C}^2$$
- Permeability of free space,
$$\mu_0 = 4\pi \cdot 10^{-7} \text{ T} \cdot \text{m/A}$$
- Magnetic constant,
$$\frac{\mu_0}{4\pi} = 1 \cdot 10^{-7} \text{ (T} \cdot \text{m)/A}$$
- 1 atmospheric pressure,
$$1 \text{ atm} = 1.01 \cdot 10^5 \text{ N/m}^2 = 1.01 \cdot 10^5 \text{ Pa}$$
- Wien's displacement constant, $b = 2.9 \cdot 10^{-3}$ m $\cdot$ K
- Stefan-Boltzmann constant,
$$\sigma = 5.67 \cdot 10^{-8} \text{ W/m}^2/\text{K}^4$$

# Theory Question 1 [10 marks]

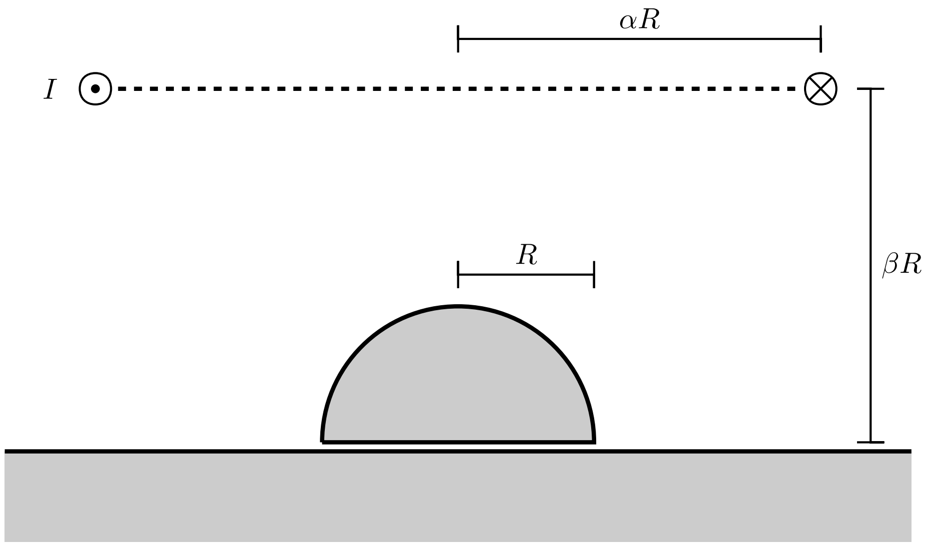
This problem consists of two independent parts.

## A. Magnetosphere [5 marks]

Iron is a *soft ferromagnetic* material, meaning that it has relative permeability $\mu_r \gg 1$ and zero intrinsic magnetisation.

**(A.1)** Show that the magnetic field at the surface of an iron body is approximately perpendicular to the surface. [0.5 pts]

An iron hemisphere of radius $R$ sits face down on a very large iron plane. Assume that there is a small air gap between the plane and the bottom of the hemisphere (with relative permeability 1). A thin conducting loop of radius $\alpha R$ is centered a distance $\beta R$ above the hemisphere and has current $I$ flowing through it.




**(A.2)** Find the net magnetic force on the hemisphere. [3 pts] *Give your answer in terms of the integral*

$$J(k) = \int_0^{2\pi} \frac{\cos(\phi)}{(k - \cos(\phi))^{3/2}}\, d\phi.$$

**(A.3)** Find numerically the maximum value of $\alpha$ such that the hemisphere can be lifted, assuming that $\beta$ and $I$ can be chosen freely. [0.5 pts]

**(A.4)** Find an equation for the maximum value of $\beta$ such that the hemisphere can be lifted, assuming that $\alpha$ and $I$ can be chosen freely. [1 pt]

## B. World Eater [5 marks]

According to general relativity, a particle's distance $r$ from a black hole of mass $M$ obeys:

$$\frac{\mathrm{d}^2 r}{\mathrm{d}\tau^2} = -\frac{GM}{r^2} + \frac{\ell^2}{r^3} - \frac{3GM\ell^2}{c^2 r^4}$$

Here, $\tau$ is the particle's proper time and $\ell$ is its conserved angular momentum per unit mass around the black hole.

A black hole of mass $M$ travels at speed $v$ through a medium with density $\rho$, accreting mass as it moves. Near the black hole, the gravitational field is so strong that you may treat the medium as a fluid of noninteracting particles. Assume that $(\rho G^3 M^2)^{1/6} \ll v \ll c$.

**(B.1)** Find the rate of increase of the black hole's mass. [1.5]

**(B.2)** Find the drag force on the black hole, neglecting the medium's gravitational interaction with itself. Qualitatively explain your result. [2]

**(B.3)** Including the medium's gravitational interaction with itself, give an asymptotic estimate for the drag force. [1.5]

# Theory Question 2 [10 marks]

This problem consists of two independent parts.

## A. Efficient Lighting [5.5 marks]

Let $S$ be a two-dimensional, simply connected room with perfectly reflecting walls, illuminated by omnidirectional light sources located in the room's interior. We strictly work in the geometric optics regime and neglect all reflections off of non-differentiable corners. Sources do not affect light passing through them.

**(A.1)** For any given configuration of $n \geq 2$ distinct point sources, construct $S$ such that the parts of the room illuminated by each source are disjoint, and the total illuminated area is equal to the area of $S$. Sketch two concrete examples of your construction that differ in more than a trivial sense. [1 pt]

**(A.2)** Now, for a finite number $n \geq 2$ sources which are finite, disjoint and simply connected 2D regions, whose shape and placement are **of your choice**, construct $S$ satisfying the conditions of part (a). Sketch your construction of $S$ and indicate the chosen source shape and placement. [2 pts]

**(A.3)** Likewise, for any **arbitrary** $n \geq 2$ sources which are finite, disjoint and simply connected 2D regions, describe how to generally construct $S$ satisfying the conditions of part (a). Sketch one possible example of your construction, differing in more than a trivial sense from your answer to part (b). [2.5 pts]

## B. Gas Gas Gas [4.5 marks]

**(B.1)** Consider a viscous gas cloud of finite extent orbiting around a dominating central mass. Prove that no steady state or periodic solution exists. Do not assume any specific equation of state (i.e., relation between $P$ and $\rho$). [4.5 pts] *Note: Solutions which do not demonstrate the non-existence of local minima will receive no points. Solutions which prove the result in the case of a disk will receive half points.*

# Theory Question 3 [10 marks]

## On Ice [10 marks]

A skater is skating on ice on a dry day with ambient temperature $T_0$. The skater has mass $M = 60$ kg and the skates have total area $A_{skate} = 5 \times 10^{-4}$ m$^2$ and length $L = 10$ cm in contact with the ice. The skater is moving forward at a velocity $v = 1\frac{\text{m}}{\text{s}}$. You may assume throughout the question that the stakes are perfectly vertical.

The goal of this question is to consider the physical effects that affect skating.

**(A.1)** Estimate the melting point of the ice directly under the skate in terms of the given quantities, the density of water at $0°C$ $\rho_w = 0.9998\frac{\text{g}}{\text{cm}^3}$, the density of ice at $0°C$ $\rho_i = 0.9168\frac{\text{g}}{\text{cm}^3}$, the latent heat of fusion for water $L_f = 334\frac{\text{kJ}}{\text{kg}}$, and $T_m = 273.15$ K is the temperature at which ice melts under standard conditions. [0.8 pts]

**(A.2)** In what regime do you expect the melting of the ice due to the pressure of the blade to be significant? [0.2 pts]

For part (B), we assume that $T_0$ lies within the regime where the effect we considered in (A) is significant. Do **not** make this assumption for the remainder of the question.

Aside from the pressure of the blade, intermolecular forces also play a role to create meltwater at the ice surface. The force between a molecule of water vapour and a molecule of ice with a liquid layer separating them can be modelled by

$$U(r) = \frac{C}{r^6} \tag{1}$$

where $C$ is a constant determining the strength of the intermolecular forces, and $r$ is the separation between the two molecules.

**(B.1)** Considering only the interaction between the vapour and ice layers, find the potential energy per unit area of a layer of liquid layer of height $h$, defining any quantities you need and stating any simplifying assumptions made. [1 pt]

**(B.2)** Estimate the equilibrium thickness of the liquid layer $h$ due to intermolecular forces. You are given the strength of interaction $C = 5.35 \times 10^{-74}$ J m$^6$, the molar mass of water $\mu = 18\frac{\text{g}}{\text{mol}}$, the latent heat of vaporisation of water $L_v = 2260\frac{\text{kJ}}{\text{kg}}$, and atmospheric pressure $P_0 = 1 \times 10^5$ Pa. [1.4 pts]

**(B.3)** Estimate the thickness of the liquid layer caused by pressure melting. The heat capacity of ice is $c = 2.09\frac{\text{kJ}}{\text{kg K}}$ and the thermal conductivity of ice is $\kappa = 2.18\frac{\text{W}}{\text{m K}}$. When does each effect dominate? [0.8 pts]

**(B.4)** Based on these answers, estimate the frictional force experienced by the skater near $0°C$. The viscosity of water is $\nu = 1.79 \times 10^{-3}\frac{\text{kg}}{\text{m s}}$. What does this tell you about the effects governing the frictional force experienced by the skater? [0.4 pts]

We now consider the correction caused by the frictional heat generated while the blade is sliding. This heat is conducted into the ice, which melts some of it. In previous sections we have assumed that the thickness of the liquid layer is constant across the length of the blade, but this is not the case once friction is taken into account. In this part, you may assume that the blade glides on top of the liquid layer.

**(C.1)** By considering the power dissipated by friction in shear flow in the liquid layer, write down an expression for the rate of change of the thickness along the blade length, i.e. as the ice spends more time in contact with the blade. [0.4 pts]

**(C.2)** Assume that as the blade comes into contact with the ice, an infinitesimal component of the ice is immediately molten and heated to $0°C$. By considering the vertical heat transfer into the ice block, find the rate of change of thickness along the blade length due to heat conduction into the ice. [1.4 pts]

**(C.3)** Combining the two effects, find an expression relating the thickness of the liquid layer $h(x)$ with the distance along the blade $x$. How does this compare to the effect considered in part B? [0.8 pts]

**(C.4)** Find an expression for the total frictional force experienced by the blade. How does this scale at low and high temperatures? [0.6 pts]

**(C.5)** Using this model, explain why we cannot skate on wax at room temperature. [0.4 pts]

In previous sections we have considered that the blade remains on top of the liquid layer. However, this is not in reality the case. In this section we consider the corrections caused by the effect of the blade sinking into the liquid layer.

**(D.1)** Consider a stationary blade which is sinking into a water layer of height $h$ at a rate of $\dot{h}$. Neglecting the vertical pressure variation, any obstacles to the side of the blade, and the fluid movement along the blade, find $p(y, z)$, the pressure field as a function of the vertical coordinate and the lateral coordinate. [1 pt]

**(D.2)** Find the condition for the lateral flow to have a small effect on the height profile. Find the minimum velocity for this criteria to be satisfied. [0.8 pts]